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10MAT41

Fourth Semester B.E. Degree Examination, Dec.2014/Jan.2015
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks:100

Note: ***1. Answer any FIVE full questions, selecting atleast TWO questions from each part.***
2. Use of statistical table is permitted.

PART – A

1 a. Employ Taylor's series method to find an approximate solution to find y at x = 0.1 given $\frac{dy}{dx} = x - y^2$, y(0) = 1 by considering upto fourth degree term. **(06 Marks)**

b. Solve the following by Euler's modified method $\frac{dy}{dx} = \log(x+y)$, y(0) = 2 to find y(0.4) by taking h = 0.2. **(07 Marks)**

c. Given $\frac{dy}{dx} = x^2(Hy)$ and y(1) = 1, y(1.1) = 1.233, y(1.2) = 1.548, y(1.3) = 1.979. Evaluate y(1.4) by Adams-Bash forth method. Apply corrector formula twice. **(07 Marks)**

2 a. Solve $\frac{dy}{dx} = 1 + xz$ and $\frac{dz}{dx} = -xy$ for x = 0.3 by applying Runge Kutta method given y(0) = 0 and z(0) = 1. Take h = 0.3. **(06 Marks)**

b. Use Picard's method to obtain the second approximation to the solution of $\frac{d^2y}{dx^2} - x^3\frac{dy}{dx} - x^3y = 0$ given y(0) = 1, y'(0) = 0.5. Also find y(0.1). **(07 Marks)**

c. Apply Milne's method to compute y(0.4) given y'' + xy' + y = 0, y(0) = 1, y'(0) = 0, y(0.1) = 0.995, y'(0.1) = -0.0995, y(0.2) = 0.9802, y'(0.2) = -0.196, y(0.3) = 0.956 and y'(0.3) = -0.2863. **(07 Marks)**

3 a. Derive Cauchy-Riemann equation in Cartesian form. **(06 Marks)**

b. Find an analytic function f(z) whose real part is $\frac{\sin 2x}{\cosh 2y - \cos 2x}$ and hence find its imaginary part. **(07 Marks)**

c. If f(z) is a holomorphic function of z, then show that $\left\{\frac{\partial}{\partial x}|f(z)|\right\}^2 + \left\{\frac{\partial}{\partial y}|f(z)|\right\}^2 = |f'(z)|^2$. **(07 Marks)**

4 a. Discuss the transformation $w = z + \frac{1}{z}$. **(06 Marks)**

b. Find the BLT which maps the points z = 1, i, -1 to w = i, 0, -i. Find image of $|z| < 1$. **(07 Marks)**

c. Evaluate $\int_C \left\{\frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)}\right\} dz$ where 'C' is circle $|z| = 3$. **(07 Marks)**

PART – B

5 a. Express $f(x) = x^4 + 3x^3 - x^2 + 5x - 2$ interms of Legendre polynomials. **(06 Marks)**

b. Obtain the solution of $x^2y'' + xy' + (x^2 - x^2)\,y = 0$ interms of $J_n(x)$ and $J_{-n}(x)$. **(07 Marks)**

c. Derive Rodrique's formula $P_n(x) = \frac{1}{2^n \lfloor n} \frac{d}{dx^n}[(x^2-1)^x]$. **(07 Marks)**

6 a. State the axioms of probability. For any two events A and B, prove that, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. **(06 Marks)**

b. A box 'A' contains 2 white and 4 black balls. Another box 'B' contains 5 white and 7 black balls. A ball is transferred from the box A to the box B. Then a ball is drawn from the box B. Find the probability that it is white. **(07 Marks)**

c. In a certain college 4% of the boys and 1% of girls are taller than 1.8m. Further more 60% of the students are girls. If a student is selected at random and is found to be taller than 1.8m, what is the probability that the student is a girl? **(07 Marks)**

7 a. The probability density of a continuous random variable is given by $p(x) = y_0 e^{-|x|}, -10 < x < \infty$. Find y_0. Also find the mean. **(06 Marks)**

b. Obtain the mean and variance of binomial distribution. **(07 Marks)**

c. In a test on 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average life of 2040 hours and SD of 60 hours. Estimate the number of bulbs likely to burn for.

i) More than 2150 hours.

ii) Less than 1950 hours.

iii) More than 1920 hours but less than 2160 hours.

Given A(1.5) = 0.4332, A(1.83) = 0.4664, A(2) = 0.4772. **(07 Marks)**

8 a. In a city 'A' 20% of a random sample of 900 school boys had a certain slight physical defect. In another city B, 18.5% of a random sample of 1600 school boys had the same defect. Is the difference between the proportions is significant? Why? **(06 Marks)**

b. A manufacturer claimed that atleast 95% of the equipment which he supplied to a factory conformed to specifications. An examination of a sample of 200 pieces of equipment revealed that 18 of them were faulty. Test his claim at a significance level of 1% and 5%. **(07 Marks)**

c. A set of five similar coins is tossed 320 times and the result is

No. of heads	0	1	2	3	4	5
Frequency	6	27	72	112	71	32

Test the hypothesis that the data follow a binomial distribution $\left[\chi^2_{0.05} = 11.07 \text{ for } 5df\right]$. **(07 Marks)**

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10CV42

Fourth Semester B.E. Degree Examination, Dec.2014/Jan.2015
Concrete Technology

Time: 3 hrs. Max. Marks:100

Note: *1. Answer FIVE full questions, selecting at least TWO questions from each part.*
2. Assume any missing data suitably.
3. Use of IS10262-2009 is permitted.

PART – A

1 a. Discuss briefly the ingredients of Portland cement along with their functions. **(10 Marks)**
b. What are Bouge's compounds? Explain their role in setting and hardening process of cement. **(10 Marks)**

2 a. Describe briefly fineness modulus of sand and how it is determined in the laboratory. **(10 Marks)**
b. List various tests conducted on coarse aggregate and explain any two of them. **(10 Marks)**

3 a. What are the ingredients of cement concrete? Explain their functions. **(10 Marks)**
b. List and explain the factors that affect the workability of concrete. **(10 Marks)**

4 a. Explain different types of curing of concrete. **(10 Marks)**
b. What are chemical admixtures? List the admixtures used in concrete. **(10 Marks)**

PART – B

5 a. Explain the factors affecting the strength of hardened concrete. **(10 Marks)**
b. Explain (i) Aggregate cement bond strength (ii) Flexural strength of concrete. **(10 Marks)**

6 a. Explain the factors affecting the modulus of elasticity of concrete and relation between modulus of elasticity and strength. **(10 Marks)**
b. Explain creep and shrinkage of concrete. **(10 Marks)**

7 a. List and explain the factors affecting durability of concrete. **(10 Marks)**
b. Explain chloride attack on concrete and its effect on durability of concrete. **(10 Marks)**

8 Design the concrete mix as per IS code procedure for the following data:
a. Characteristic compressive strengths at 28 day 25 MPa
b. Maximum size of aggregate : 20 mm
c. Degree of workability compaction factor 0.8 (CF)
d. Degree of quality control – Good
e. Types of exposure – Mild
f. Sp. gravity of cement – 3.15
Sp. gravity of C.A. – 2.72
Sp. gravity of F.A. – 2.60
Sand – Zone - III
Assume any other data suitably. **(20 Marks)**

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10CV43

Fourth Semester B.E. Degree Examination, Dec.2014/Jan.2015
Structural Analysis – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, selecting atleast TWO questions from each part.***

PART – A

1 a. Define the following:
   i) Linear and non-linear systems.
   ii) Geometric and material non-linearity. **(06 Marks)**

b. Explain the principle of minimum potential energy. **(05 Marks)**

c. Explain determinate and indeterminate structures with suitable examples. **(05 Marks)**

d. Find the statical and kinematic indeterminancy for the following structures: **(04 Marks)**

Fig.Q.1(d)(i)

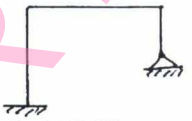

Fig.Q.1(d)(ii)

2 a. Determine the rotation and deflection at the free end of a cantilever beam shown in Fig.Q.2(a), by moment area method. Take EI = constant. **(10 Marks)**

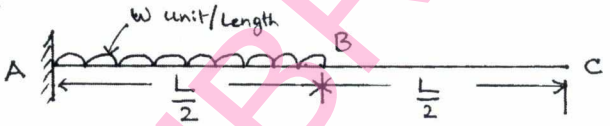

Fig.Q.2(a)

b. Determine the maximum slope and deflection for the given simply supported beam as shown in Fig.Q.2(b), by conjugate beam method. **(10 Marks)**

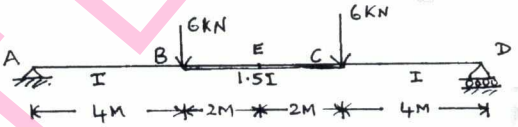

Fig.Q.2(b)

3 a. State Clark Maxwell's law of reciprocal deflection. **(02 Marks)**

b. Determine the vertical deflection at 'C' for the beam shown in Fig.Q.3(b), by Castigliano's method. **(09 Marks)**

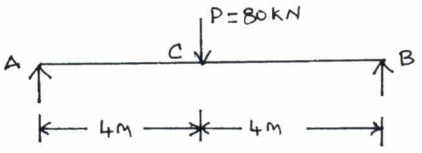

Fig.Q.3(b)

c. Determine the rotation at the free end of a cantilever beam shown in Fig.Q.3(c), by Castigliano's method. **(09 Marks)**

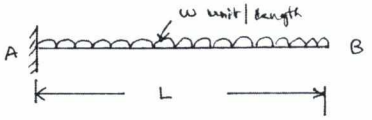

Fig.Q.3(c)

4 a. Find the vertical deflection at 'C' for the beam shown in Fig.Q.4(a), by strain energy method. **(08 Marks)**

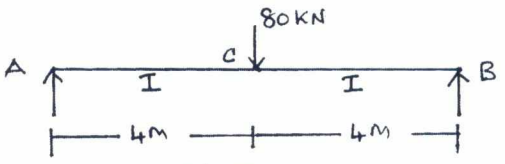

Fig.Q.4(a)

b. Determine the vertical deflection at 'C' for the frame loaded as shown in Fig.Q.4(b), by using unit load method. Cross sectional area of horizontal, vertical and inclined members are 1500, 2000 and 4000mm^2 respectively. Take EI = 2.1×10^5 N/mm^2. **(12 Marks)**

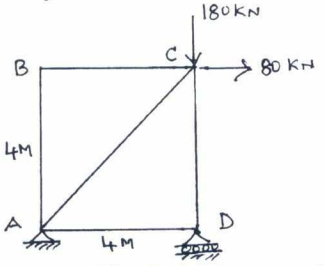

Fig.Q.4(b)

PART – B

5 A three hinged parabolic arch of span 24m, rise 6m with hinges at abutments and at crown point. Arch is subjected to a point load of 50kN and 150kN at a distance of 8m and 20m from left support. Determine:

i) Reaction at supports.
ii) Resultant reaction and its inclination at supports.
iii) Bending moments at load points and draw BMD.
iv) Normal thrust and radial shear at a distance of 6m from left and right supports.

(20 Marks)

6 A beam AB of span 4m is fixed at A and B carries a point load of 5kN at a distance of 1m from end A. Calculate the support reactions by the method of consistent deformation. Also draw BMD and SFD. Take EI = constant. **(20 Marks)**

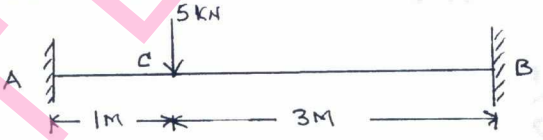

Fig.Q.6

7 Analyze the continuous beam by three moment equation. Find reactions and draw BMD and SFD. **(20 Marks)**

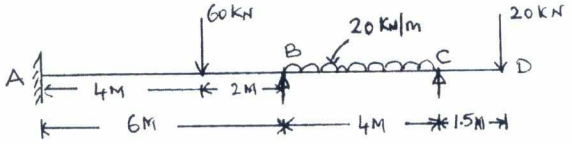

Fig.Q.7

8 A parabolic two hinged arch has a span of 32m and a rise of 8m. A uniformly distributed load of 1kN/m covers 8m horizontal length of the left side of the arch. If I = $I_0 \sec\theta$, where θ is the inclination of the arch of the section to the horizontal, and I_0 is the moment of inertia of the section at the crown. Find out the horizontal thrust at hinges and bending moment at 8m from the left hinge. Also, find out normal thrust and radial shear at this section. **(20 Marks)**

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10CV44

Fourth Semester B.E. Degree Examination, Dec.2014/Jan.2015
Surveying – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer FIVE full questions, selecting at least TWO questions from each part.***

PART – A

1 a. Explain the following terms with reference to a theodolite (i) Transiting (ii) Swinging (iii) Line of collimation (iv) Centering (v) Vertical axis. **(10 Marks)**

b. Describe the method of measuring horizontal angle by repetition method. What are the errors that are eliminated by repetition method? **(10 Marks)**

2 a. What are the fundamental lines of a transit theodolite? State the derived relationships between them. **(10 Marks)**

b. Explain the object, necessity, test and adjustment of making the horizontal axis perpendicular to the vertical axis of a theodolite by "SPIRE TEST". **(10 Marks)**

3 a. Explain the method of finding the reduced level of the top of the given object, when base is inaccessible by double plane method. **(10 Marks)**

b. In order to ascertain the elevation of the top (Q) of the signal on a hill, observations were made from two instrument stations P and R at a horizontal distance 100 meters apart, the stations P and R being in line with Q. The angles of elevation of Q at P and R were 28° 42′ and 18° 6′ respectively. The staff reading upon benchmark of elevation 287.280 were respectively 2.870 and 3.750 when the instrument was at P and R, the telescope being horizontal. Determine the elevation at the foot of the signal if the height of the signal above its base is 3 meters. **(10 Marks)**

4 a. With usual notation, derive the distance and elevation formulae for staff vertical and line of sight inclined upwards in fixed hair method of tacheometric surveying. **(10 Marks)**

b. A tacheometer was setup at a station 'A' and the reading on a vertically held staff at B were 2.255, 2.605 and 2.955. The line of sight being at an inclination of +8° 24′. Another observation on the vertically held staff at B.M gave the readings 1.640, 1.920 and 2.200, the inclination of the line of sight being +1° 6′. Calculate the horizontal distance between A and B, and the elevation of B if the RL of BM is 418.685 meters. The constants of the instruments were 100 and 0.3. **(10 Marks)**

PART – B

5 a. Explain the method of setting out a simple curve by Rankine's method of deflection angles. **(10 Marks)**

b. Two tangents intersect at a chainage of 1000 mt, the deflection angle being 28°. Calculate the necessary data to setout a simple curve of Radius 250 mt by Rankine's method and tabulate the results. Peg interval = 20 mt; Least count of theodolite = 20″. **(10 Marks)**

6 a. A compound curve consisting of two simple circular curves of radii 350 mt and 500 mt is to be laidout between two straights T_1I and IT_2. PQ is the common tangent, at point of compound curvature, D. The angles IPQ and IQP are respectively 55° ad 25°. Sketch and calculate the tangent points T_1I and IT_2. **(10 Marks)**

b. From an eccentric station S, 12.25 meters to the west of the main station B, the following angles were measured: $\underline{BSC} = 76° 25' 32''$

$\underline{CSA} = 54° 32' 20''$

The stations S and C are to the opposite sides of the line AB, calculate the correct angle ABC if the lengths AB and BC are 5286.5 and 4932.2 mt respectively. **(10 Marks)**

7 a. What is a Transition curve? List the functions and conditions to be fulfilled by a transition curve. **(10 Marks)**

b. A road bend which deflects 80° is to be designed for a maximum speed of 100 kmph, a maximum centrifugal ratio of ¼ and a maximum rate of change of acceleration 30 cm/sec^2, the curve consists of a circular arc combined with two cubic spirals. Calculate : i) the radius of the circular arc ii) the requisite length of transition iii) the total length of the composite curve and iv) chainages of beginning and end of transition curve, and of the functions of the transition curve with the circular arc, if the chainage of PI is 42862 meters. **(10 Marks)**

8 a. The following perpendicular offsets were taken at 10 mt intervals from a survey line to an irregular boundary line.

3.25, 5.60, 4.20, 6.65, 8.75, 6.20, 3.25, 4.20, 5.65

Calculate the area enclosed between the survey line, the irregular boundary line and the first and last offsets, by the application of (i) average ordinate rule (ii) Trapezoidal rule and (iii) Simpson's rule. **(10 Marks)**

b. A road embankment is 10 mt wide with side slopes ½ to 1. Assuming the ground to be level in a direction transverse to the centre line, calculate the volume contained in a length of 120 meters, the centre heights at 20 m intervals being in meters

2.2, 3.7, 3.8, 4.0, 3.8, 2.8, 2.5 **(10 Marks)**

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10CV45

Fourth Semester B.E. Degree Examination, Dec.2014/Jan.2015
Hydraulics and Hydraulic Machines

Time: 3 hrs. Max. Marks:100

Note: ***1. Answer any FIVE full questions, selecting atleast TWO questions from each part.***
2. Missing data may suitably assumed.

PART – A

1 a. The efficiency η of a fan depends on density ρ, dynamic viscosity μ of the fluid, angular velocity ω, diameter D of the rotor and discharge Q. Express η as

$$\eta = f\left[\frac{Q}{\omega D^3}, \frac{\mu}{\rho \omega D^2}\right].$$ **(08 Marks)**

b. Derive different scale ratio's as per Reynold's model law. **(06 Marks)**

c. A flow meter tested in the laboratory, gave a pressure drop of 200kN/m^2 for a discharge of 0.2m^3/s in 200mm diameter pipe. If a geometrically similar model is tested in 1000mm diameter pipe at identical conditions of fluid, determine the corresponding discharge and pressure drop in the model. **(06 Marks)**

2 a. Distinguish between: Pipe flow and open channel flow. **(06 Marks)**

b. Derive the Chezy's equation for uniform flow in open channel with usual notations. **(07 Marks)**

c. A canal is to have a trapezoidal section with one side vertical and the other sloping at 60° to the horizontal. It has to carry water at 30m^3/s with mean velocity 2 m/s. Compute the dimensions of the section which will require minimum lining. **(07 Marks)**

3 a. Define specific energy. Explain specific energy curve (sketch). **(06 Marks)**

b. A horizontal rectangular channel 4m wide carries a discharge of 16m^3/s. If the initial depth of flow is 0.5m, determine is there a possibility of formation of hydraulic jump? If the jump forms, determine the sequent depth, Froude number after jump and energy loss. **(06 Marks)**

c. Give the classification of surface profiles in case of GVF. **(08 Marks)**

4 a. Show that the efficiency of a jet striking a series of flat vanes mounted on the periphery of a circular wheel is maximum when the jet velocity is double of vane velocity and maximum efficiency is 50%. **(10 Marks)**

b. A jet of 30mm radius strikes normally on a fixed plate, with a velocity of 35m/s. Calculate the force exerted by the jet on the plate. If the plate is moving with 15 m/s in the direction of the plate, find the efficiency of the jet. **(10 Marks)**

PART – B

5 a. Derive an equation of force exerted by a jet on an unsymmetrical curved vane tangentially, when vane K moving in the x-direction. Draw the velocity triangles and explain. Also find the workdone and efficiency. **(10 Marks)**

b. A jet of water with velocity 40m/s strikes a curved vane, which is moving with velocity 20m/s. The jet makes an angle of 30° with the direction of motion of vane at inlet and leaves at an angle of 90° to the direction of motion of vane at outlet. Draw the velocity triangles at inlet and outlet and determine the vane angles at inlet and outlet so that the water enters and leaves the vane without shock. **(10 Marks)**

6 a. Give the classification of turbine with examples. **(10 Marks)**

b. Design a Pelton wheel turbine required to develop 1471.5 kW power under a head of 160m at 420 rpm. The overall efficiency may be taken as 85%. Assume $c_v = 0.98$, $c_u = 0.46$, jet ratio = 12. **(10 Marks)**

7 a. Define draft tube. What are its functions? **(06 Marks)**

b. What is cavitation? How to eliminate it? **(06 Marks)**

c. A Kaplan turbine runner is to be designed to develop brake power of 7350kW, under a head of 5.5m. Diameter of bass is $1/3^{rd}$ of diameter of runner. Assuming speed ratio = 2.09, flow ratio = 0.68, calculate: i) diameter of runner and boss; ii) speed of runner. Take $\eta_0 = 85\%$. **(08 Marks)**

8 a. Define:
i) Manometric efficiency
ii) Mechanical efficiency
iii) Overall efficiency. **(06 Marks)**

b. Derive an expression for minimum starting speed of a centrifugal pump. **(06 Marks)**

c. The internal and external diameters of the impeller of a centrifugal pump are respectively 200mm and 40mm. The pump is running at 1200rpm. The vane angles of the impeller at inlet and outlet are 20° and 30°. Water enters radially and velocity of flow is constant. Determine the workdone by the impeller per unit weight of water. **(08 Marks)**

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10CV46

Fourth Semester B.E. Degree Examination, Dec.2014/Jan.2015
Building Planning and Drawing

Time: 4 hrs. Max. Marks:100

Note: ***1. Q. No. ONE is compulsory.***
2. Answer any TWO full questions form Part-B.
3. Assume the missing data suitably.

PART – A

1 Draw plan at sill level, front elevation and sectional elevation taking section which gives max details (section AA). Write the schedule of opening. The details are as follows for the line diagram shown in Fig.Q.1.

- 3 coarses size stone masonry of 200mm thick in BBM with CM(1:8).
- Plinth concrete of 50mm thick.
- BBM wall of 230mm thick in CM 1:6
- Height of storey 3.1M
- Floor level is 300mm above GL provide steps with T = 250mm, R = 150mm.
- Lintel is 2.1m above GL.
- Slab is 100mm thick made of M20 grade concrete.
- Provide standard dimensions for lintel, parapet and bed concrete.
- The details of opening are as follow

$D_1 = 1.0m \times 2.1m$
$D_2 = 0.9m \times 2.0m$
$W_1 = 1.5m \times 1.2m$
$W_2 = 1.2m \times 1.2m$
$W_3 = 1.0m \times 1.2m$
$V_1 = 0.5 \times 0.4m$
$D_3 = 1.2m \times 2.0m$

a. Plan **(25 Marks)**
b. Elevation **(15 Marks)**
c. Section **(15 Marks)**
d. Schedule of opening **(05 Marks)**

PART – B

2 Draw the following to a suitable scale.

a. Plan of column **(02 Marks)**
b. Plan of foundation showing reinforcement details. **(06 Marks)**
c. Longitudinal section through column and footing. **(12 Marks)**

Size of column (400 × 600)mm
Height of column above ground level – 3500mm
Depth of foundation – 1200mm, 400mm in rectangular portion and slopes from 400mm to 800mm from rectangular portion to junction of column and footing. Use M20 grade concrete and Fe415 steel. Size of footing is (1600 × 1800)mm with 150mm concrete column consists of 4 No. of 16 # bars as longitudinal reinforcement, 8# @ 180mm c/c as transverse reinforcement. Footing mat consists of 12# @ 200 mm c/c both ways.

3 Develop the plan for a primary health centre with provision for
i) Doctor's consulting room; ii) First aid room; iii) Treatment room; iv) Front office; v) Medical store; vi) laboratory; vii) Store room; viii) WC for men and women.
Provide standard dimensions. Prepare plan and also bubble diagram. **(20 Marks)**

4 Prepare water supply and sanitary layout for the residential building shown in Fig.Q.1 with standard symbols and notations. **(20 Marks)**

5 Draw a single leaf fully panelled door for (1.20m × 2.10m) providing
Bottom rail – (250 × 40)mm
Lock rail – (150 × 40)mm
Styles – (120 × 40)mm
Frame – (150 × 40)mm
Top rail – (100 × 40)mm **(20 Marks)**

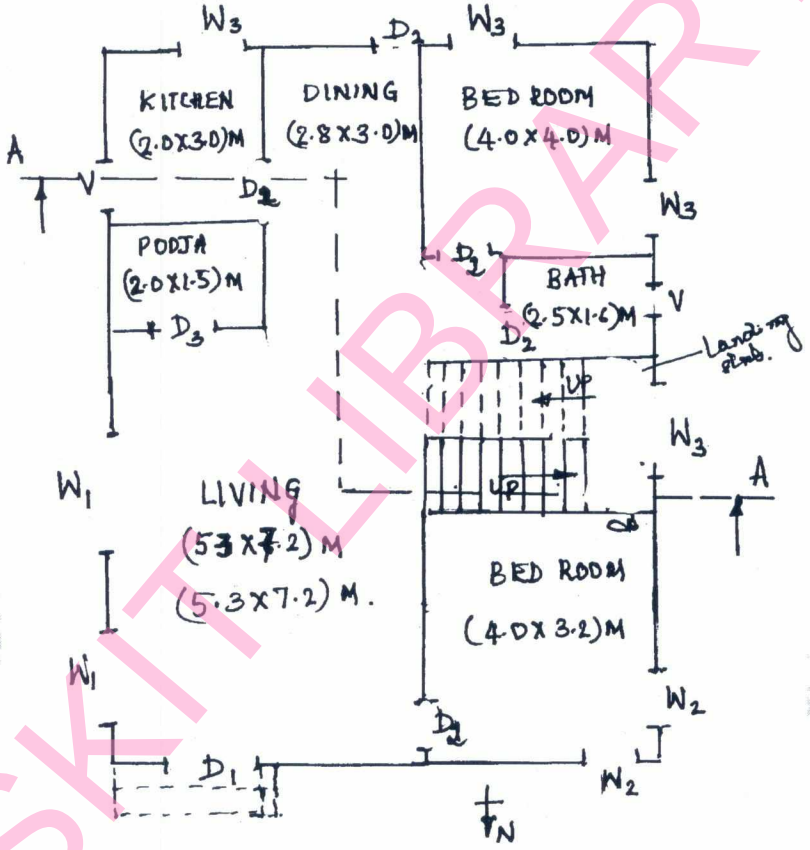

Fig.Q.1

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MATDIP401

Fourth Semester B.E. Degree Examination, Dec.2014/Jan.2015
Advanced Mathematics – II

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. If l, m, n are the direction cosines of a line then prove that $l^2 + m^2 + n^2 = 1$ **(06 Marks)**

b. Find angle between any two diagonals of a cube. **(07 Marks)**

c. Find angle between two lines whose direction cosines satisfy the equations, $l + m + n = 0$ and $2l + 2m - mn = 0$. **(07 Marks)**

2 a. With the usual notations derive the equation of the plane in the form $lx + my + nz = 0$. **(06 Marks)**

b. Find the equation of the plane through (1, 2, -1) and perpendicular to the planes $x + y - 2z = 5$ and $3x - y + 4z = 12$. **(07 Marks)**

c. Find the shortest distance between the lines,

$$\frac{x-6}{3} = \frac{y-7}{-1} = \frac{z-4}{1} \text{ and}$$

$$\frac{x}{-3} = \frac{y+9}{2} = \frac{z-2}{4}$$

(07 Marks)

3 a. Prove that $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b}(\vec{c}.\vec{a}) - \vec{c}(\vec{a}.\vec{b})$. **(06 Marks)**

b. Find the sine of angle between the vectors $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$. **(07 Marks)**

c. Show that the vectors $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + \hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i} + 4\hat{j} - \hat{k}$ are coplanar. **(07 Marks)**

4 a. Find the unit normal vector to the space curve $\vec{r} = 4\sin t\,\hat{i} + 4\cos t\,\hat{j} + 3t\,\hat{k}$. **(06 Marks)**

b. A particle moves along the curve $\vec{r} = \cos 2t\,\hat{i} + \sin 2t\,\hat{j} + t\,\hat{k}$. Find the velocity and acceleration at $t = \frac{\pi}{8}$ along $\sqrt{2}\,\hat{i} + \sqrt{2}\,\hat{j} + \hat{k}$. **(07 Marks)**

c. Find angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x = z^2 + y^2 - 3$ at (2, -1, 2) **(07 Marks)**

5 a. Find the directional derivative of x^2yz^3 at (1, 1, 1) in the direction of $\hat{i} + \hat{j} + 2\hat{k}$. **(06 Marks)**

b. If $\vec{F} = (x + y + 1)\hat{i} + \hat{j} - (x + y)\hat{k}$ then show that $\vec{F}.\text{curl}\,\vec{F} = 0$ **(07 Marks)**

c. Show that the vector $\vec{F} = (3x^2 - 2yz)\hat{i} + (3y^2 - 2zx)\hat{j} + (3z^2 - 2xy)\hat{k}$ is irrotational. **(07 Marks)**

6 a. Prove that $L[\sin at] = \frac{a}{s^2 + a^2}$. **(05 Marks)**

b. Find $L[\sin t \sin 2t \sin 3t]$. **(05 Marks)**

c. Find $L[te^{-t} \sin 2t]$. **(05 Marks)**

d. Find $L\left[\frac{e^{at} - e^{bt}}{t}\right]$. **(05 Marks)**

7 a. If $L[f(t)] = \int_0^\infty e^{-st} f(t)dt$ then prove that $L[f''(t)] = s^2 L[f(t)] - sf(0) - f'(0)$. **(05 Marks)**

b. Find $L^{-1}\left[\frac{s+2}{s^2 - 4s + 13}\right]$. **(05 Marks)**

c. Find $L^{-1}\left[\frac{s+1}{(s-2)^3}\right]$. **(05 Marks)**

d. Find $L^{-1}\left[\log\left(\frac{s-a}{s-b}\right)\right]$. **(05 Marks)**

8 a. Using Laplace transform solve $y'' - 2y' + y = e^{2t}$ with $y(0) = 0$, $y'(0) = 1$. **(10 Marks)**

b. Using Laplace transform solve the simultaneous equation,

$$\frac{dx}{dt} + y = \sin t$$

$$\frac{dy}{dt} + x = \cos t$$

given that x(0)=1, y(0) = 0 **(10 Marks)**

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